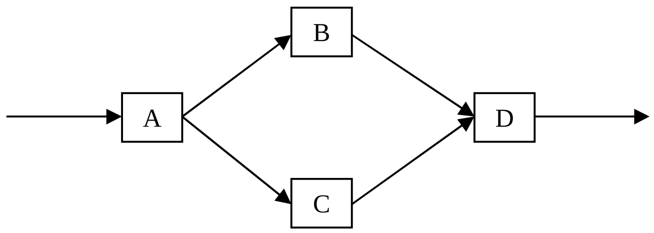
UNIT 8. ESTIMATION OF ABSOLUTE PERFORMANCE & COMPUTER SYSTEM SIMULATION

Output analysis is the examination of data generated by a simulation. Its purpose is either to predict the performance of a system or to compare the performance of two or more alternative system designs. Here, we will discuss estimating the value of one or more system performance measures, which is called as absolute performance of a system.

8.1 TYPES OF SIMULATION WITH RESPECT TO OUTPUT ANALYSIS

There are two types in analyzing simulation output –

- **Terminating (or transient) Simulations:** A terminating simulation is one that runs for some duration of time T_E, where E is a specified event (or set of events) that stops the simulation. Such a simulated system opens at time 0 under certain initial conditions and closed at the stopping time T_E. For example –
  - A bank opens at 8.30am (time 0) with no customers present and 8 of the 11 counters working (initial conditions), and closes at 4.30pm (Time T_E = 480 minutes). Here, the event E is that, the bank was open for 480 minutes. The simulation analyst is interested in modeling the interaction between the customers and counters over the entire day.
  - A communication system consists of several components as shown below. Consider the system over a period of time T_E, until the system fails. The stopping event E is happens when (i) A fails or (ii) D fails or (iii) both B and C fails.



- **Steady – State Simulations:** A non-terminating or a steady-state system is a system that runs continuously or at least over a very long period of time. For example, telephone lines, internet etc. A simulation of non-terminating system starts at simulation time 0 under initial conditions defined by the analyst and runs for some analyst-specified period of time T_E. Normally, the analyst wants to study the system when it is not being influenced by initial conditions of the model at time 0. For example –
  - Consider a large web-based order-processing company having many customers worldwide. It has a huge computer system with many servers, workstations, and peripherals working 24x7. An analyst would like to study the peak work-load of this system and the tolerance of additional work during peak working hours.

8.2 ABSOLUTE MEASURES OF PERFORMANCE AND THEIR ESTIMATION

Consider the estimation of a performance parameter θ (or Φ) of a simulated system. It is desired to have a point estimate and an interval estimate of θ (or Φ). The length of the interval estimate is the measure of the error in the point estimate. The simulation output data may be of two types –

- **Discrete – time data:** If the simulation data are of the form {Y1, Y2, …. Yn}, it is called as discrete time data. For example, Y_i may be the delay of customer *i*, total cost in the week *i* etc. The notation θ is used as a parameter for discrete data, and θ is nothing but a mean of the given data.
- **Continuous – time data:** If the simulation data are of the form {Y(t), 0≤ t ≤ T_E }, then it is called as continuous data. For example, Y(t) may be queue length at time *t*. The notation Φ is used as a parameter for continuous data, which is referred as a time-weighted mean.

The estimation of θ and Φ are discussed in following sections.

8.2.1 Point Estimation

The point estimator of θ, based on the data {Y1, Y2, … Yn} is defined as –

$$\hat{\theta} = \frac{1}{n}\sum_{i=1}^{n} Y_i$$

Here, $\hat{\theta}$ is a sample mean based on the sample of size *n.* The pointer estimator $\hat{\theta}$ is said to be unbiased for θ, if

$$E\left(\hat{\theta}\right) = \theta$$

In general, they are not equal. And, $E\left(\hat{\theta}\right) - \theta$ is called as the ***bias*** in the pointer estimator $\hat{\theta}$.

It is better to have an unbiased estimator, or an estimator with a small bias.

8.2.2 Confidence – Interval Estimation

Confidence – interval estimation is generally used for continuous distributions. Suppose, the model represented by random variable *Y* is normal distribution with mean θ and variance σ^2. Now,

$$\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$$

The estimate is computed based on the sample, and it has an error. A confidence interval is a measure of that error. Let the sample variance be –

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(Y_i - \bar{Y}\right)^2$$

Now, the confidence interval is –

$$\bar{Y} \pm t \frac{S}{\sqrt{n}}$$

Here, *t* is the *t-distribution* value with n-1 degrees of freedom.

8.3 OUTPUT ANALYSIS FOR STEADY – STATE SIMULATIONS

Consider a single run of a simulation model whose purpose is to estimate a steady-state or long-run characteristic of the system. Suppose, the single run produces the observations *Y1, Y2, …* , which are auto-correlated samples. The steady-state measure of performance θ for discrete data is defined as –

$$- = \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} Y_i$$

The steady-state performance for a continuous-time output measure *{Y(t), t≥0}* is defined as –

$$\text{¤} = \lim_{T_E \to \infty} \frac{1}{T_E} \int_0^{T_E} Y(t)\,dt$$

In both of the above cases, the probability is 1.

The sample size *n* and the simulation time T_E is a design chose, and decided by the nature of the problem. The simulation analyst will chose the simulation run length (*n* or T_E) with several considerations in mind –

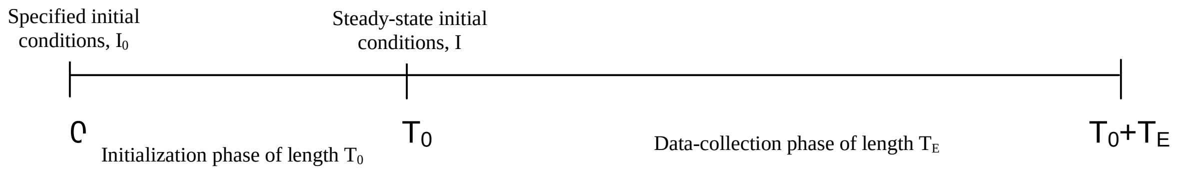
- Any bias in the point estimator that is due to artificial or arbitrary initial conditions.
- The desired precision of the point estimator as measure by the standard error or confidence interval half-width.
- Budget constraints on the time available to execute the simulation.

8.3.1 Initialization Bias in Steady – State Simulation

There are several methods of reducing the point-estimator bias caused by using artificial and unrealistic initial conditions in a steady-state simulation.

- The first method is to initialize the simulation in a state that is more representative of long-run conditions. This method is called as ***intelligent initialization.*** For example, placing customers in a queue and in service for queuing simulation, having some components fail or degrade in a reliability simulation etc. There are two ways to specify initial conditions intelligently –
    - o If the system exists, collect its data and use the same to specify initial conditions.
    - o Obtain initial conditions from a second model of the system that has been simplified enough to make it mathematically solvable.

- The second method to reduce impact of initial conditions is to divide each simulation run into two phases: (i) Initialization phase from time 0 to T_0 (ii) data collection phase from time T_0 to stopping time $T_0 + T_E$ as shown below –



The choice of T_0 is important, because, the system state at the beginning I_0 should be nearer to steady-state conditions *I.* Also, the length T_E of data-collection phase should be long enough to guarantee precise estimates of steady-state behavior.

Important Questions on this Unit:

1. What are transient and steady state simulations with respect to output analysis? Explain with an example each.
   OR
   Explain the types of simulation model with respect to output analysis.

2. Explain the concept of point estimation and confidence interval estimation.
   OR
   Which are the measures of performance of a simulated system? How do you estimate them?

3. Explain initialization bias in steady state simulation.
4. Briefly explain output analysis for steady state simulation.